Numerical Integration

August 2017

1. Find the values of the following integrals. You may use either the trapezium method as outlined in the tutorial or the `scipy.quad` function. If you use the trapezium method, check that the error in your result is negligible.

    a. $\int_1^2 x \log x \, dx$

    b. $\int_{-\pi}^{\pi} \frac{dx}{x^6 + 1}$

    c. $\int_1^2 x^x \, dx$

    d. $\int_0^\infty \frac{\sin(x)}{x} \, dx$

2. Plot solutions to the following differential equations in the domain [-1,1]. Assume an initial condition $x(-1) = 1, x'(-1) = -1$ for each equation.

    a. $\frac{d^2 x}{dt^2} + \sin t = \frac{dx}{dt}$

    b. $\frac{d^2 x}{dt^2} + \sin x = \frac{dx}{dt}$

    c. $\frac{d^3 x}{dt^3} + \frac{d^2 x}{dt^2} = tx,\ x''(-1) = 2$

3. On separate graphs, plot $x(t)$ and $y(t)$ in the domain [-1,1] if they satisfy the following system of differential equations:

    $\frac{dx}{dt} \frac{dy}{dt} = xy,\ \frac{dy}{dt} = x \frac{dx}{dt},\ x(-1) = 0,\ y(-1) = 1$