Numerical Integration

August 2017

1. Find the values of the following integrals. You may use either the trapezium method as outlined in the tutorial or the `scipy.quad` function. If you use the trapezium method, check that the error in your result is negligible.

   a. $\int_1^2 x \log x dx$

```
from math import log
from scipy.integrate import quad
def f(x):
    return x*log(x)
print(quad(f,1,2)[0])
```

```
## 0.6362943611198906
```

   b. $\int_{-\pi}^{\pi} \frac{dx}{x^6+1}$

```
from math import pi
from scipy.integrate import quad
def f(x):
    return 1./(x**6+1)
print(quad(f,-pi,pi)[0])
```

```
## 2.0930886145206866
```

   c. $\int_1^2 x^x dx$

```
from scipy.integrate import quad
def f(x):
    return x**x
print(quad(f,1,2)[0])
```

```
## 2.050446234534731
```

   d. $\int_0^\infty \frac{\sin(x)}{x} dx$

```
from math import sin
def f(x):
    return 1 if x==0 else sin(x)/x
def trap(func,n,a,b):
    h = (b-a)/n
    intgr = 0.5 * h * (func(a) + func(b))
    for i in range(1,int(n)):
        intgr = intgr + h*func(a+i*h)
    return intgr

#initial values
a,b=0,1e6
n=1e6 #number of points
#compare this integral with a larger range, finer grid
#repeat until it converges
while(abs(trap(f,n,a,b)-trap(f,n*4,a,b*2)) > 1e-6):
  n*=2
  b*=2
  print("increased n,b to {0},{1}".format(n,b))
```

```
print(trap(f,n,a,b))
#from scipy.integrate import quad
#print(quad(f,a,b)[0]) -> fails
```

```
## 1.5707954694386728
```

2. Plot solutions to the following differential equations in the domain [-1,1]. Assume an initial condition $x(-1) = 1, x'(-1) = -1$ for each equation.

   a. $\frac{d^2x}{dt^2} + \sin t = \frac{dx}{dt}$

```
from scipy import *
from scipy.integrate import odeint
from pylab import *
from rmdplot import figsize,savefig

def equations_a(u,t):
    x,x_prime = u
    return (x_prime, x_prime - sin(t)) #vector (x',x'')

t = linspace(-1,1,2001)
u0=array([1,-1]) #x,x'

part_a = odeint(equations_a,u0,t)
figure(figsize=figsize)
plot(t, part_a[:,0],'k')
savefig("figure/parta.pdf",caption="2.a")
```

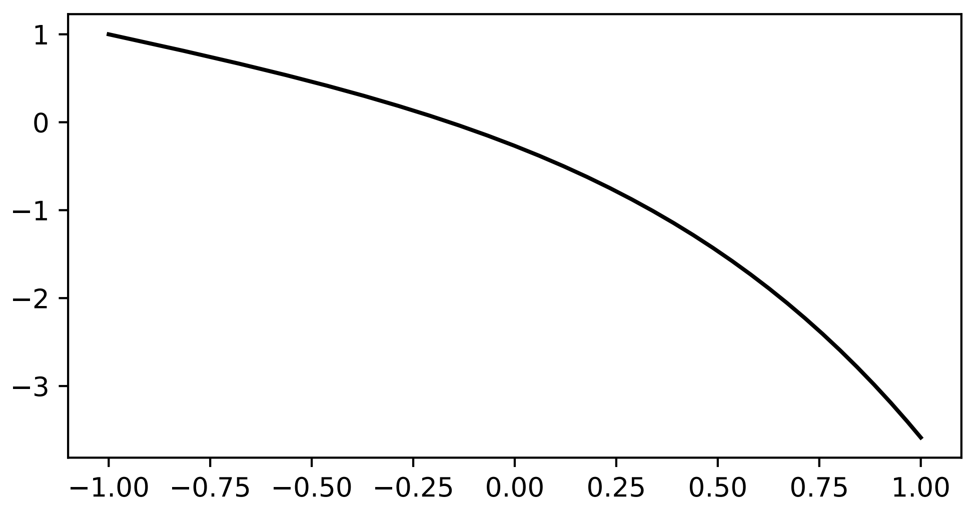

Figure 1: 2.a

   b. $\frac{d^2x}{dt^2} + \sin x = \frac{dx}{dt}$

```
from scipy import *
from scipy.integrate import odeint
from pylab import *
from rmdplot import figsize,savefig
```

```python
def equations_b(u,t):
    x,x_prime = u
    return (x_prime, x_prime - sin(x)) #vector (x',x'')

t = linspace(-1,1,2001)
u0=array([1,-1]) #x,x'

part_b = odeint(equations_b,u0,t)
figure(figsize=figsize)
plot(t, part_b[:,0],'k')
savefig("figure/partb.pdf",caption="2.b")
```

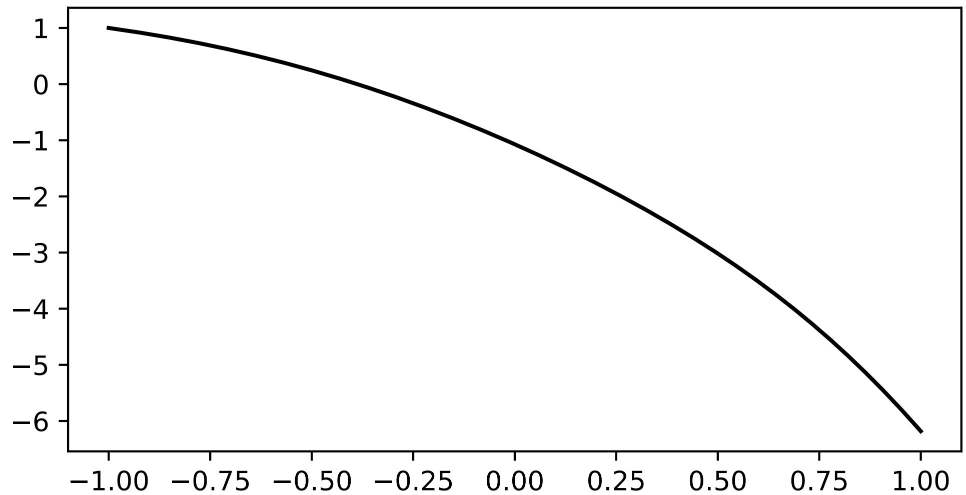

Figure 2: 2.b

c. $\frac{d^3x}{dt^3} + \frac{d^2x}{dt^2} = tx$, $x''(-1) = 2$

```python
from scipy import *
from scipy.integrate import odeint
from pylab import *
from rmdplot import figsize,savefig

def equations_c(u,t):
    x,x_prime, x_prime_prime = u
    return (x_prime, x_prime_prime, t*x-x_prime_prime) #vector (x',x'',x''')

t = linspace(-1,1,2001)
u0=array([1,-1,2]) #x,x',x''

part_c = odeint(equations_c,u0,t)
figure(figsize=figsize)
plot(t, part_c[:,0],'k')
savefig("figure/partc.pdf","2.c")
```

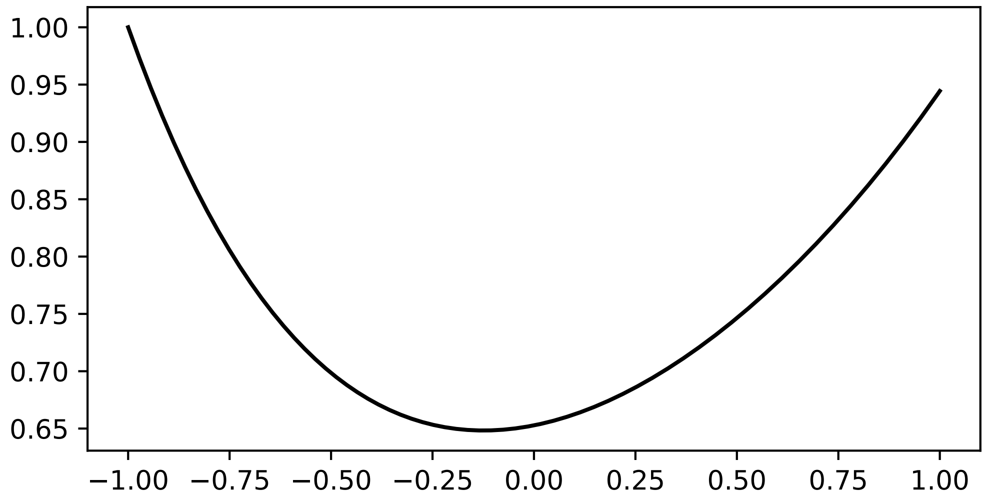

Figure 3: 2.c

3. On separate graphs, plot $x(t)$ and $y(t)$ in the domain [-1,1] if they satisfy the following system of differential equations:

   $\frac{dx}{dt}\frac{dy}{dt} = xy$, $\frac{dy}{dt} = x\frac{dx}{dt}$, $x(-1) = 0$, $y(-1) = 1$

````
```python
from scipy import *
from scipy.integrate import odeint
from pylab import *
from rmdplot import figsize,savefig

def equations(u,t):
    x, y = u
    return (sqrt(y), x * sqrt(y)) #the vector (x', y')

t = linspace(-1,1,2001)
u0 = array([0, 1])   #initial values of x and y
u = odeint(equations, u0, t)
figure(figsize=figsize)
subplot(121)
plot(t, u[:, 0])
title('X')
subplot(122)
plot(t, u[:, 1])
title('Y')

savefig("figure/ni3.pdf","3")
```


![3](figure/ni3.pdf)
````