Functions and Modules Questions

1. Write a function sum that takes two integer parameters a and b and returns the sum of squares of all the integers n such that a <= n <= b.

2. Write a function triangle that takes three real parameters which represent the lengths of three sides of a triangle. The function should return True if the triangle is right-angled and False otherwise.

3. a. Write a function factorial that takes a nonnegative integer parameter and returns its facorial; we define 0! = 1.

   b. The number e can be defined as the limit of the infinite sum 1/0! + 1/1! + 1/2! + 1/3! + 1/4! + ... Write a function compute_e which takes a positive integer parameter. This parameter indicates the number of terms that are to be summed (i.e. compute_e(3) should produce the sum 1/0! + 1/1! + 1/2!). Sum the terms and return the approximate value of e. Call/use the function you wrote in part a in your solution.

4. You have already written a program that calculates all prime numbers less than 100. Write a function primes that takes one integer parameter and returns the number of primes less than that integer.

5. Write a function solve_quadratic that takes three real parameters a, b, c (in that order). The function should return the roots of the quadratic equation ax**2 + bx + c = 0 as a tuple. If the roots are imaginary, the function should print "No real solutions" and return nothing.

6. Write a function plot_polynomial that takes two integers and a list as parameters. The list contains the coefficients of the polynomial (with the first corresponding to the coefficient of the term of the highest degree). The two integers are the bounds of the domain of the polynomial. The function should produce a graph of the polynomial.

7. Write a function lissajous that takes four real parameters a, b, c, d. Let a be the amplitude of oscillation in the x direction, b be the angular frequency of oscillation in the x direction, c be the amplitude of oscillation in the y direction, d be the angular frequency of oscillation in the y direction. Assume the oscillations are in phase and described by cosine waves. The function should display the Lissajous figure for this motion on the time interval [0, 1000].

8. A quick introduction to recursive functions. A function that calls itself is called recursive. Here is an example of recursion to calculate a factorial, which you should have already done using a loop:

   ```
   def factorial(n):
           if n <= 1: return 1
           else: return n * factorial(n-1)
   ```

Here we have a *base case* (n = 0 or n = 1) and the function calls itself to reduce any other case to the base case. Write a function sum() that takes a single integer parameter n. It should return the sum of all the positive integers up to and including n (i.e. sum(5) = 1 + 2 + 3 + 4 + 5 = 15). Use recursion.