Numerical Integration Questions

1. Find the values of the following integrals. You may use either the trapezium method as outlined in the tutorial or the *scipy.quad* function. If you use the trapezium method, check that the error in your result is negligible.
    a. $\int_1^2 xlogxdx$
    b. $\int_{-\pi}^{\pi} \frac{dx}{x^6+1}$
    c. $\int_1^2 x^x\, dx$
    d. $\int_0^{\infty} \frac{sinxdx}{x}$

2. Plot solutions to the following differential equations in the domain [-1, 1]. Assume an initial condition $x(-1) = 1, x'(-1) = -1$ for each equation.
    a. $\frac{d^2x}{dt^2} + sint = \frac{dx}{dt}$
    b. $\frac{d^2x}{dt^2} + sinx = \frac{dx}{dt}$
    c. $\frac{d^3x}{dt^3} + \frac{d^2x}{dt^2} = tx, x''(-1) = 2$

3. On separate graphs, plot $x(t)$ and $y(t)$ in the domain [-1, 1] if they satisfy the following system of differential equations:

$$\begin{cases} \frac{dx}{dt}\frac{dy}{dt} = xy \\ \frac{dy}{dt} = x\frac{dx}{dt} \end{cases}, x(-1) = 0, y(-1) = 1$$